Maxwell's Equations

The Foundation of Electromagnetism

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Outline

Introduction to Maxwell's Equations

- Maxwell's equations are a set of four fundamental equations that describe how electric and magnetic fields interact
- They form the foundation of classical electromagnetism
- Unified electricity, magnetism, and light
- Predicted the existence of electromagnetic waves
- Published by James Clerk Maxwell in 1865

Gauss's Law for Electricity

Differential Form

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Integral Form

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$$

- Relates electric field to electric charge
- Electric field lines originate from positive charges
- Electric field lines terminate on negative charges

Gauss's Law for Magnetism

Differential Form

$$\nabla \cdot \mathbf{B} = 0$$

Integral Form

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

- There are no magnetic monopoles
- Magnetic field lines always form closed loops
- Net magnetic flux through any closed surface is zero

Faraday's Law of Induction

Differential Form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Integral Form

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{A}$$

- Changing magnetic field induces electric field
- Basis for electric generators and transformers
- Lenz's law: induced current opposes the change

Ampère-Maxwell Law

Differential Form

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Integral Form

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \int_S \mathbf{E} \cdot d\mathbf{A}$$

- Electric current and changing electric field create magnetic field
- Maxwell's addition of displacement current term was crucial
- Predicts electromagnetic waves

Physical Meaning of Each Equation

1. **Gauss's Law (E)**: Electric charges create electric fields
2. **Gauss's Law (B)**: No magnetic charges exist
3. **Faraday's Law**: Changing magnetic fields create electric fields
4. **Ampère-Maxwell Law**: Electric currents and changing electric fields create magnetic fields

The equations show the intimate relationship between electric and magnetic fields, leading to the concept of electromagnetic waves.

Wave Equation

Starting from Maxwell's equations in vacuum ($\rho = 0$, $\mathbf{J} = 0$):

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{1}$$

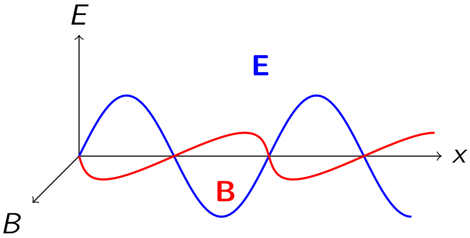
$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \tag{2}$$

This is the wave equation with wave speed:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \text{ m/s}$$

- Predicted speed of light matches experimental value
- Unified light with electromagnetism

Properties of EM Waves



- Transverse waves
- E and B fields perpendicular to each other and to direction of propagation
- In phase
- Travel at speed of light in vacuum

Applications of Maxwell's Equations

Technology:

- Radio and TV broadcasting
- Mobile communications
- Radar systems
- Microwave ovens
- Fiber optics
- Electric motors
- Transformers

Science:

- Understanding light
- Electromagnetic spectrum
- Antenna design
- Circuit theory
- Plasma physics
- Astrophysics
- Quantum electrodynamics

Summary

- Maxwell's equations are the foundation of classical electromagnetism
- They describe how electric and magnetic fields interact
- Four equations:
  - Gauss's Law for E
  - Gauss's Law for B
  - Faraday's Law
  - Ampère-Maxwell Law
- Predict electromagnetic waves traveling at speed of light
- Unified electricity, magnetism, and optics
- Essential for modern technology and physics

Thank You

Questions?